LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2018**

16/17/18PMT1MC03 – ORDINARY DIFFERENTIAL EQUATIONS

Date: 30-10-2018 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

Answer all questions. Each question carries 20 marks.

1. (a) If u and v are solutions of $L(y(t)) = a_0(t)y'' + a_1(t)y' + a_2(t)y = 0$, prove that $W[u,v] = ke^{-\int\frac{a_1(t)}{a_0(t)}dt}$ where k is a constant. (5)

(OR)

(b) Prove that $x(t) = c_1\, x_1(t) + c_2\, x_2(t)$ is a solution of $L(y(t)) = 0$ where $x_1(t)$ and $x_2(t)$ are any two solutions of $L(y(t)) = 0$ and c_1, c_2 are any two constants. (5)

(c) Find the general solution of the equation $x'''(t) - x'(t) = cost$, using the method of variation of parameters. (15)

(OR)

(d) Assume that $a_0(t), a_1(t), \dots, a_n(t)$ and $b(t)$ are real valued continuous functions of t defined on the interval I of the real line R and that $a_0(t) \neq 0$ for all $t \in I$. Prove that the initial value problem $a_0(t)x^{(n)} + a_1(t)x^{(n-1)} + \cdots + a_n(t)x = b(t)$ with initial conditions $x(t_0) = \alpha_1$, $x'(t_0) = \alpha_2$, $\dots, x^{(n-1)}(t_0) = \alpha_n$ admits one and only one solution. (15)

2. (a) State and prove Rodrigue's formula. (5)

(OR)

(b) Prove that (i) $P_l(1) = 1$, and (ii) $P_l'(1) = l(l+1)/2$. (5)

(c) Use Frobenius method to solve $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + (3 - x)y = 0$. (15)

(OR)

(d) Obtain the generating function for the Legendre polynomials. (15)

3. (a) State and prove any two recurrence relations for Bessel's function. (5)

(OR)

(b) When n is a non-zero integer, show that $J_{-n}(x) = (-1)^n J_n(x)$. (5)

(c) Solve: $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$ where $n \geq 0$. (15)

(OR)

(d) State and prove the integral representation of Bessel's function. (15)

4. (a) For the distinct parameters λ and μ , let x and y be the corresponding solutions of the Strum-Liouville problem respectively such that $[pW(x,y)]_A^B = 0$. Prove that $\int_A^B r(s)\, x(s)\, y(s)\, ds = 0$. (5)

(OR)

(b) Using the method of successive approximations, solve the initial value problem $x'(t) = -x(t)$, $x(0) = 1$, $t \geq 0$. (5)

(c) Let $G(t, s)$ be the Green's function. Prove that $x(t)$ is a solution of the equation $L(x(t)) + f(t) = 0$, $a \leq t \leq b$, if and only if $x(t) = \int_a^b G(t, s) f(s)\, ds$. (15)

(OR)

(d) State and prove Picard's theorem for boundary value problem. (15)

5. (a) Discuss the stability behavior of the system $x' = -x$ at origin. (5)

(OR)

(b) Show that the null solution of the equation $x' = A(t)x$ is stable if and only if there exists a positive constant k such that $|\phi(t)| \leq k, t \geq t_0$. (5)

(c) State and prove the two fundamental theorems on the stability of non-autonomous systems. (15)

(OR)

(d) Discuss the stability of linear system $x' = Ax$ using Lyapunov's function. (15)
